Eur. Phys. J. B **7**, 523–525 (1999)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

Rapid Note

Evanescent modes are not necessarily Einstein causal

G. Nimtz[a]

II. Physikalisches Institut, Universität Köln, Zülpicherstrasse 77, 50937 Köln, Germany

Received: 12 December 1998 / Accepted: 14 December 1998

Abstract. Previously, experiments with microwave signals have shown that evanescent modes can travel faster than light. Several theoretical investigations have proven that in the case of signals with unlimitedly high frequency components, such superluminal velocities do not violate Einstein causality, thus group, signal, and energy velocities are $\le c$ where c is the vacuum velocity of light. In this letter I shall show that frequency band *limitation* is a fundamental property of signals and that such signals containing only evanescent modes can violate Einstein causality.

PACS. 03.65.-w Quantum mechanics – 03.50.De Maxwell theory: general mathematical aspects – 03.30.+p Special relativity

1 Introduction

Evanescent modes have an imaginary wave number and represent the wave mechanical tunneling analogy [1,2]. They are found in the process of optical total internal reflection, in undersized waveguides, and in forbidden frequency bands of periodic dielectric hetero-structures. Various experiments with microwave signals have revealed superluminal velocities of evanescent modes [3–5]. See reference [6] for a recent conference on this subject. In the case of unlimited frequency bands the high-frequency components of the signals are not evanescent (*i.e.* they do not tunnel in the wave mechanical picture, their energy being higher than the potential barrier). These high energy components form a front which travels with the velocity of light c and cannot be overtaken by the low-frequency superluminal evanescent modes. The exponential attenuation of the tunneling components results in a pulse reshaping as displayed in Figure 1 [7]. The evanescent components of the signals are shifted to earlier pulse arrival time, they have traveled faster than light without overtaking the luminally traveling front thus not violating Einstein causality [8].

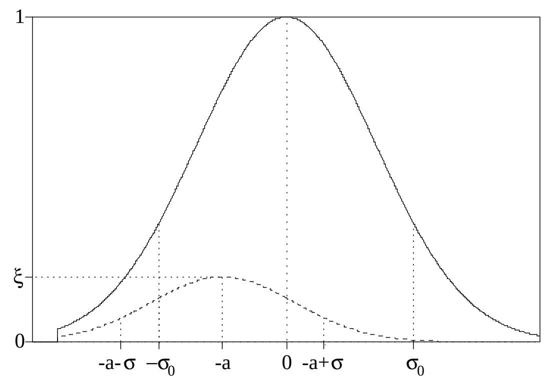

Fig. 1. Comparison of normalized intensity *vs.* time of an airborne signal (solid line) and a tunneled signal (dotted line) moving from right to left. Both signals have a sharp step at their beginning and the frequency spectrum is infinite. The tunneled signal is reshaped, attenuated, and its maximum has traveled at superluminal speed but both fronts have traversed the same distance in the same time with the light velocity c. Here ξ is the maximum of the tunneled pulse, a is the shift of the maximum, σ is the variance of the tunneled signal, and σ_0 is the variance of the incoming pulse.

2 Signals

Some properties of signals are introduced with the example of a modern amplitude modulated (AM) signal. Such an AM signal is displayed in Figure 2. The halfwidths of the pulses represent the information conveyed, *i.e.* the number of bits. I want to remind the reader that a signal has to be independent of its magnitude. The relative frequency band width of this signal is only 10^{-4}. The signal is glass-fibre guided and has an infrared carrier with a wavelength of 1.5 μm. In theory switching on a signal generates infinitely high frequencies. Such an example is displayed in Figure 1. However, signals with an infinite spectrum are impossible, since Planck has shown in 1900 that the minimum energy of a frequency component is $\hbar\omega$,

[a] e-mail: g.nimtz@uni-koeln.de

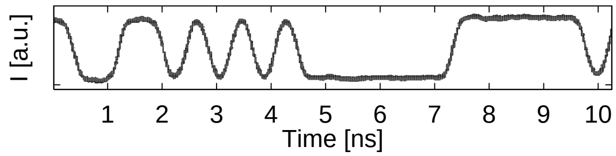

Fig. 2. Intensity I *vs.* time of a frequency band limited AM-signal of a glass-fibre guided infrared carrier. The plotted intensity is the envelope of the squared amplitude of the high frequency infrared wave. The pulses' halfwidths represent the number of bits, *i.e.* the information.

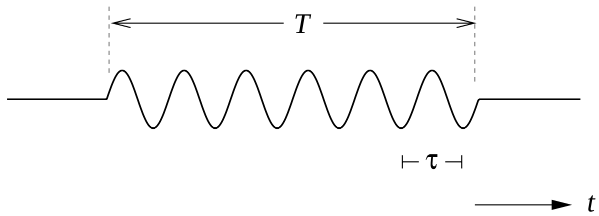

Fig. 3. A frequency band unlimited sinusoidal signal with T/τ oscillations.

where $\hbar$ is the Planck constant and ω is the angular frequency. Since a signal has a finite energy (may be as small as of the order of 100 photons only) it follows that its spectrum has also to be finite. In classical physics, however, $\hbar \rightarrow 0$ holds. A classical detector would be able to measure infinitely small amounts of energy and a signal with a finite energy can have an infinite spectrum. This is in contradiction to quantum mechanics and to any physical detector. A detector needs at least one quantum $\hbar\omega$ and with $\omega \rightarrow \infty$ the signals energy would become infinite. This has the consequence that frequency limitation is a fundamental property of physical signals.

3 An evanescent mode signal

The fact that signals are frequency band limited is an important point, which has not been given much attention until recently [5,9,10]. However, this fundamental property may have serious consequences in the case of evanescent modes. In the following I shall present results on the signal propagation of evanescent modes following the procedure of Sommerfeld and Brillouin [11].

In order to investigate the wave propagation they assumed a sinusoidal signal with the angular frequency $\omega_0 = 2\pi/\tau$ which is terminated at both ends, as shown in Figure 3. Such a wave form is composed of two unterminated waves, one beginning at $t = 0$ and the second at $t = T$ with opposite phase, so that the two cancel for all time $t > T$:

$$f(t) = \begin{cases} 0 & (t < 0) \\ \sin \omega_0 t & (0 < t < T). \\ 0 & (T < t) \end{cases}$$

The Fourier analysis of this signal yields after several transformations [11]

$$f(t) = \frac{1}{2\pi} \Re \int_{-\infty}^{+\infty} \left[e^{i\omega(t-T)} - e^{i\omega t} \right] \frac{d\omega}{\omega - \omega_0}. \quad (1)$$

If this signal traverses a distance z each wave ω propagates with its phase velocity $v_{ph}(\omega)$ and the integral becomes

$$f(t,z) = \frac{1}{2\pi} \Re \int_{-\infty}^{+\infty} \left[e^{i\omega(t-T-z/v_{ph}(\omega))} - e^{i\omega(t-z/v_{ph}(\omega))} \right] \frac{d\omega}{\omega - \omega_0}. \quad (2)$$

In any dispersive medium the highest frequency components will arrive at z with speed c. They do not interact with the medium and their weak oscillations are called forerunners or front. At a lower speed the main signal will arrive, which will be deformed. If it is possible to determine the exact moment when this main signal arrives, this defines the signal velocity. One has to distinguish between the wavefront velocity, which might be determined by a forerunner, and the colloquial signal velocity, with which the main part of the wave propagates in a dispersive medium. In general, the signal velocity measured depends on the sensitivity of the detecting apparatus used.

If we are not dealing with a signal with a sudden start and a sudden ending (the realistic and the only procedure by which signals and reactions are mediated) we can suppress frequencies very different from ω_o and the formula becomes by expansion of the exponents [11]

$$f(t,z) = \frac{1}{2\pi} \Re \Big\{ e^{i\omega_0(t-z/v_{ph}(\omega_0))} \times \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} \left[e^{i(\omega-\omega_0)(t-T-z/v_{gr}(\omega_0))} - e^{i(\omega-\omega_0)(t-z/v_{gr}(\omega_0))} \right] \frac{d\omega}{\omega - \omega_0} \Big\}. \quad (3)$$

This equation represents a signal beginning progressively at $t = 0$ and arriving at z at a time $t = z/v_{gr}$, ending at $t = T$ and $t = T + z/v_{gr}$, respectively. The velocity of the wave front is now equal to the group velocity $v_{gr}(\omega_0) = d\omega/dk$.

This holds for a complex wave number k. In an evanescent medium the wave number κ is purely imaginary, assuming to be independent of frequency in the range $\omega_0 \pm \Delta\omega$, it follows that

$$f(t,z) = \frac{1}{2\pi} e^{-\kappa z} \Re \left\{ e^{i\omega_0 t} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} \left[e^{i(\omega-\omega_0)(t-T)} - e^{i(\omega-\omega_0)t} \right] \frac{d\omega}{\omega - \omega_0} \right\}. \quad (4)$$

This frequency band limited signal decays exponentially in traversing the distance z, however, without spending any time in the evanescent region. The observed time delay is due to the phase shift at the barrier boundary independent of the length z of the tunneled distance. Even the rising edge of the signal propagates with the group velocity $> c$, no matter how large that may become in traversing the evanescent region. An amazing result, which, however, is in agreement with the experimental data.

4 Conclusion

In this letter I have shown that a signal has a finite spectrum. This is in consequence of the finite energy content of any signal and of the quantization of radiation. For frequency band limited signals conveyed by superluminal evanescent modes the classical property of the Helmholtz and of the Maxwell equations becomes evident. The limited spectrum is not due to a technical deficiency of signal generators [10].

There is another dilemma due to frequency band limitation of signals like the real one presented in Figure 2. The Fourier transform of a frequency band limited signal is unlimited in time. Thus there would be no start and no end although they are measured, see Figure 2. The explanation of this amazing phenomenon is that the intensity outside the observed signal's time is too small to be measurable, the corresponding energy is smaller than a quantum $\hbar\omega$.

According to equation (4) signals composed only of evanescent modes do not spend time in the evanescent region [3,12,13]. As long as the transmission dispersion can be neglected in the limited spectrum, a significant pulse reshaping does not take place and signals as well as the evanescent energy can travel faster than light. The latter has been shown also with single photons [14]. This behaviour is quite different from the example shown in Figure 1 where the spectrum was unlimited and the luminal front of the airborn signal was not overtaken by the tunneled signal, only the maximum and the center of mass have traveled superluminally. Evanescent signals can travel at superluminal speeds according to equation (4). Thus an observer communicating with light velocity may see a change of chronological order of cause and effect. Standing behind the tunnel one may see the signal leaving the tunnel before entering it.

I thank Peter Mittelstaedt and Rolf Pelster for comments and suggestions.

References

1. A. Sommerfeld, *Vorlesungen über Theoretische Physik* Band IV, Optik Dieterichsche Verlagsbuchhandlung (1950).
2. R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, MA, 1969), Chap. 24-4.
3. G. Nimtz, W. Heitmann, Prog. Quantum Electronics **21**, 81 (1997).
4. A. Enders, G. Nimtz, J. Phys. I France **2**, 1693 (1992).
5. W. Heitmann, G. Nimtz, Phys. Lett. A **196**, 154 (1994).
6. *Proceedings of the international workshop on Superluminal Velocities*, June 6–10 1998, Cologne, Ann. Phys. **7**, Nr.7-8 (1998).
7. T. Emig, Diploma Thesis, University of Cologne 1995.
8. F. Low, P. Mende, Ann. Phys. NY **210**, 380 (1991); K. Hass, P. Busch, Phys. Lett. A **185**, 9 (1994); G. Diener, Phys. Lett. A **223**, 327 (1996); T. Emig, Phys. Rev. E **54**, 5780 (1996).
9. A. Ranfagni, D. Mugnai, Phys. Rev. E **52**, 11288 (1995).
10. M. Büttiker, H. Thomas, Superlattices and Microstructures **23**, 781 (1998).
11. L. Brillouin, *Wave Propagation and Group Velocity* (Academic Press, New York and London, 1960).
12. Th. Hartman, J. Appl. Phys. **33**, 3427 (1962).
13. A. Enders, G. Nimtz, Phys. Rev. E **48**, 632 (1994).
14. A. Steinberg, P. Kwiat, R. Chiao, Phys. Rev. Lett. **71**, 708 (1993).